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Model Reduction of Semi-Positive Definite Systems Reflected to Actuator and Sensor Locations

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A model reduction method for large flexible structures is developed in order to deal with rigid-body modes and to obtain higher modes related to starting vectors corresponding to multi-directional locations of actuators and sensors. The algorithm is involved with the frequency shifting technique, Krylov vector sequence and the inverse iteration method. The reduced-order model by the proposed algorithm has shown better dynamic response than the model constructed by truncated eigenvectors of a full-order system because the eigenvectors are not always the best choice in a dynamic analysis. Futhermore, the algorithm for a semi-positive definite system can accommodate a damping effect so that the efficient vectors depending on load vectors can be produced without increasing the system order and without using complex calculus, unlike the standard eigenproblem with damping effect. Numerical example is given with a flexible space structure characterized by closely spaced eigenvalues.

Key Words: Krylov Vectors, Semi-Positive Definite System, Eigenproblem, Quasi-Coordinate, Model Reduction

1. Introduction

An important issue in modeling of flexible structures or any other large-scale systems is the dimensionality of the system, especially those formulated by the finite element method. The finite element approach can lead to an accurate model, but results in a high order system. Hence, a model reduction approach plays an important role for efficient dynamic analysis and real-time controller design. In the model reduction approach, the selection of projection basis is important to the accuracy of the reduced-order model. Many authors have researched an basis selection by eigenmodes, static modes in component mode synthesis and Krylov vectors, which can be considered as static modes. In particular, Krylov vectors have been used in eigenvalue analysis and applied to structural dynamics model reduction problems. Several algorithms have been developed using the Krylov vectors as in the Wilson method (Wilson, et al., 1982) and the Lanczos algorithm (Chen and Taylor, 1988; Nour-omid and Clough, 1985).

Su and Craig (1991) presented a model reduction algorithm with the combination of Krylov vectors and a parameter matching concept which reduces the model order without destroying the symmetry and physical meaning of the damped system matrices. Furthermore, the reduced-order model has the valuable property of parameter matching. However, their algorithm cannot effectively account for specific parameters and higher frequency contents due to matching of low frequency modes. Such loads with high frequency contents may be, for example, produced by earthquakes or by common control signals and external disturbances. Skelton and Yousuff (1983) showed that certain higher mode shapes are important in the application of their modal reduction method to large flexible structures.

By including the frequency dependent vectors

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in the process of vector generation, accuracy of either dynamic analysis or control system design is dramatically improved in references (Joo, et al., 1989; Sandridge and Haftka, 1991). Xia and Humar (1992), motivated by reference (Joo, et al., 1989), presented an algorithm in order to account for the frequency content of the loading. For systems with a paramete that may strongly influence the response, particularly for loading with high frequency contents, the original Ritz vectors algorithm is improved in the sense of selecting more essential modes. However, those algorithms deal with an undamped linear system and destroy the symmetry and the physical meaning of the system matrices because it is necessary to put a second-order matrix differential equation into a first-order form.

Furthermore, the previous vector generation algorithms cannot be directly used for a semi-positive definite system which has singularity in the stiffness matrix. Craig and Bampton (1971) presented a computational procedure for identifying rigid-body and deformation coordinates by partial factorization of the stiffness matrix and for obtaining a suitable coordinates transformation which permits the rigid-body coordinates to be eliminated and a problem suitable for iteration to be formulated. Ricle (1990) employed a spectrum shift in order to remove rigid-body modes, while allowing the stiffness matrix to be factorized in developing load-dependent Ritz vectors.

By extending the frequency dependent Krylov vectors (FDKV) algorithm developed by Sung (1997), the singularity in the stiffness matrix can be automatically accommodated. The singularity arises from rigid-body modes of system dynamics for flexible spacecraft. In this development, the idea of spectrum shifting of the stiffness matrix is used with the previous FDKV algorithm developed in reference (Sung, 1997). In order to account for the rigid-body modes, the shifting strategy should be used consecutively for rigid-body mode removal by taking small spectrum shifts and for the new Krylov vector sequence update. Starting vectors are orthogonalized with respect to the rigid-body vectors. The rigid-body modes are computed by the null space of the stiffness matrix. After the generation of Krylov vectors, a set of vectors for the flexible-body modes is reorthogonalized, and then the entire set of Krylov vectors is constructed. With the modification of the algorithm developed by Sung (1997), new algorithm can account for desired parameters with low frequency as well as high frequency contents for a general second-order damped system by considering the influence of actuators and sensors.

In the following sections, the algorithm of frequency dependent Krylov vectors (FDKV) for a semi-positive definite system is presented by consecutively employing a spectrum shifting technique. A numerical simulation and comparison are given for illustration. The efficiency and accuracy of the FDKV algorithm is numerically demonstrated by the application to the SCOLE (Spacecraft COntrol Laboratory Experiment) model which is characterized by closely spaced flexible modes and 6 rigid-body modes. Quasi-Krylov equations are presented as preconceived vectors to reduce the computational burden for on-line dynamic analysis of the SCOLE system. Finally, a conclusion is made with respect to multi-dimensional systems.

2. Model Reduction for a Semi-Positive Definite System

An algorithm for model reduction is developed by strategically utilizing the spectrum shifting technique to remove the rigid-body modes and to produce vectors associated with the flexible body. The projection subspace does not destroy the symmetry and the physical meaning of the system matrices by utilizing a second-order formulation. The concept of component mode synthesis is employed for the semi-positive definite damped system. The vectors produced by the new algorithm are used as admissible vectors to approximate the system deflection. It is shown that the new admissible vectors have better accuracy than eigenvectors alone, as shown in the case of impulse response, because it is not always the best choice to select the eigenvectors as an admissible set. By taking the block form as starting vectors, which consists of several vectors, the algorithm can produce a set of vectors with each iteration. Therefore, it is suited for a multidimensional system such as large flexible structures. The analytical development is presented and summarized.

The vector second-order system with rigid-body modes is expressed as

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=Fu \quad (1)$$

$$y=Px+V\dot{x} \quad (2)$$

where $x\in\mathbf{R}^n$ is the displacement vector; $u\in\mathbf{R}^l$ the input vector; $y\in\mathbf{R}^m$ the output measurement vector; M, C, and K are the mass, damping, and stiffness matrices; P and V are the displacement and velocity sensor distribution matrices, respectively. The damping matrix is assumed to be symmetric. The displacement response vector $x(t)$ can be written as

$$x(t)=Qz(t) \quad (3)$$

where $Q=(q_1, q_2, \cdots, q_p)$ is a set of projection vectors and $z(t)=(z_1, z_2, \cdots, z_p)^T$ is a set of reduced coordinates with $p<n$.

In the conventional mode-superposition method (Bathe, 1982; Meirovitch, 1980), the selection of the projection vectors can be determined by either exact eigenvectors or a set of independent vectors through certain iterative procedures. However, the usage of exact eigenvectors for a damped system involves a complex vector basis so that the computation procedure is more complicated due to doubled order and complex calculus. Here, an iterative procedure is sought without increasing the system order and using complex computation.

The rigid-body modes can be obtained from

$$Q_r=\mathcal{N}(K) \quad (4)$$

where r and $\mathcal{N}$ stand for rigid-body and null space, respectively. As the previous development (Xia and Humar, 1992), a Krylov sequence with damping effect is represented by

$$S_{damped}=[q,\ D_\sigma q,\ D_\sigma^2 q,\ \cdots,\ D_\sigma^{s-1}q] \quad (5)$$

where q is any non-zero vector (which is commonly a static correction vector), and

$$D_\sigma=(\hat{K}-\sigma\hat{M})^{-1}\hat{M} \quad (6)$$

where σ represents the shift. In order to obtain an iterative formulation, the following equation,

$$x(t)=Qe^{(\lambda-\sigma)t} \quad (7)$$

where λ is the eigenvalue and Q the eigenvector, is substituted into each state variable of $x(t)$ in Eq. (1). As a basic relation of vector iteration method, the eigenvalue problem can be formulated as

$$(\lambda-\sigma)\begin{bmatrix} C+2\sigma M & M \\ M & 0 \end{bmatrix}\begin{Bmatrix} Q \\ (\lambda-\sigma)Q \end{Bmatrix}= \begin{bmatrix} -K-\sigma C-\sigma^2 M & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} Q \\ (\lambda-\sigma)Q \end{Bmatrix} \quad (8)$$

where the shifted matrices on both sides remain symmetric. From the above eigenproblem, an iteration formula by the inverse iteration technique is represented as

$$\begin{bmatrix} -K-\sigma C-\sigma^2 M & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} Q_{j+1}^d \\ Q_{j+1}^v \end{Bmatrix} =\begin{bmatrix} C+2\sigma M & M \\ M & 0 \end{bmatrix}\begin{Bmatrix} Q_j^d \\ Q_j^v \end{Bmatrix}. \quad (9)$$

After algebra operations, the iterative procedure for damped system can be expressed as

$$Q_{j+1}^d=[-K-\sigma C-\sigma^2 M]^{-1} \{[C+2\sigma M]Q_j^d+MQ_j^v, \quad (10)$$

$$Q_{j+1}^v=Q_j^d. \quad (11)$$

A procedure for the FDKV algorithm is presented in Table 1. Unlike the previous procedure in reference (Sung, 1997), the formula is used to generate an entire vector sequence with a more generalized procedure. Note that the new procedure becomes the Su and Craig procedure (Su and Craig, 1991) by taking zero shift for σ. The entire set of the new Krylov vectors is formed as follows:

$$x(t)=[Q_r,\ Q_f]z(t) \quad (12)$$

where Q_r and Q_f are $n\times r$ rigid-body matrix and $n\times s$ (*blocks*) flexible-body matrix, respectively. Hence, the order of the final transformation matrix Q is $p=r+s$ (*blocks*) where s will be chosen by a designer.

By developing the frequency dependent Krylov vectors for semi-positive definite systems, a desired parameter can be dynamically located. Moreover, due to block generation of vectors, the procedure can be applied to a system with closely

Table 1 FDKV algorithm for semi-positive definite system.

Operation	Calculation
rigid-body modes	$Q_r=\mathcal{N}(K)$
Starting vector:	
independent vector selection	$\hat{P}=[F, P^T, V^T, M^{-1}V^T]$
frequency shift	$K_r=K-\sigma_r M$
	$R_o^d=K_r^{-1}\hat{P}$
	$R_0^v=-M^{-1}V^T$
singular value decomposition	$U_0S_0U_0^T=\text{svd}\{(R_0^d)^T K_r R_0^d\}$
first vectors	$Q_1^d=R_0^d U_0 S_0^{1/2}$
	$Q_1^d=R_0^d U_0 S_0^{1/2}$
remove Q_r from Q_1	$Q_1^d=Q_1^d-\sum_{i=1}^{r} Q_i\{Q_i^T M Q_1^d\}$
	$Q_1^v=Q_1^v-\sum_{i=1}^{r} Q_i\{Q_i^T K_r Q_1^d\}$
frequency dependent Krylov vectors with different shifts ($\sigma=\sigma_r$ or σ_f):	For $j=2, 3, \cdots, s-1$, Iterate
	$W=[-K-\sigma C-\sigma^2 M]$
	$R_j^d=W^{-1}\{[C+2\sigma M]Q_j^d+MQ_j^v\}$
	$R_j^v=Q_j^d$
	$K_\sigma=K-\sigma M$
orthogonalization	$R_j^d=R_j^d-\sum_{j=1}^{N} Q_i^d\{(Q_i^d)^T K_\sigma R_j^d\}$
	$R_j^v=R_j^v-\sum_{i=1}^{j} Q_i^d\{(Q_j^d)^T K_\sigma R_j^d\}$
singular value decomposition	$U_jS_jU_j^T=\text{svd}\{(R_j^d)^T K_\sigma R_j^d\}$
(j+1)th vectors	$Q_{j+1}^d=R_j^d U_j S_j^{1/2}$
	$Q_{j+1}^v=R_j^v U_j S_j^{1/2}$
	End
Form the s-block projection matrix:	$Q_f=[Q_1^v, Q_1^d, Q_2^d, \cdots, Q_{s-1}^d]$
Final Matrix	$Q=[Q_r, Q_f]$

spaced eigenvalues such as large flexible space structures. Furthermore, the proposed scheme can treat complex loading cases, while the Ritz vectors (Wilson, et al., 1982) procedure can handle only single loading cases. Since the starting vectors can account for the influence of actuators and sensors, the new algorithm could be well suited for control applications. In addition, the size of the block can be reduced if the actuator/sensor allocation is collocated, since $\hat{P}$ is the linearly independent part of $[F, P^T, V^T, M^{-1}V^T]$.

In Table 1, a small value for $\sigma_r(>0)$ can be chosen to remove the singularity of the stiffness matrix and to make the first load dependent vectors converge quidcly. σ_f rad/sec can be selected to dynamically locate the load dependent vectors according to actuator location and excitation input. If the actuator location is related to certain mode shapes in a cluster, σ_f can be taken as a value in the middle of the cluster. On the other hand, if there are several predominant frequencies that are widley separated, the generation of frequency dependent Krylov vectors may be repeated with one or more additional shifts.

3. Quasi-Krylov Equations of Motion

The equations of motion for the SCOLE model are adopted from Meirovitch and Quinn (1987). The SCOLE model consists of the mast supporting the antenna which is a steel tube 10 feet (3.048 m) long. The antenna consists of 12 aluminum tubes, each 2 feet (0.6096 m) long, welded together to form a hexagonal-shaped grid. The shuttle is simulated by a steel plate of uniform thickness with a mass of 13.85 *slugs* (202.1241 Kg_m). Those equations are used for the evaluation of the model reduction algorithm. The spacecraft consists of a shuttle carrying an antenna connected to the shuttle by means of a mast, as shown in Fig. 1.

The equations of motion of a flexible spacecraft consist of six nonlinear ordinary differential equations for the rigid-body motion of a reference frame attached to the spacecraft in undeformed state, coupled with a set of partial differential equations for the vibration of the elastic members relative to the rigid frame. By utilizing a perturbation approach for a solution, the equations are separated into a set of equations for the rigid-body motions (representing zero-order effects), and a set of equations for the small elastic motions and deviations from the rigid-body motions (representing first-order effects). The model is discretized by finite element method with hermite cubic polynomial as interpolation functions for bending and linear functions for axial and torsional deformations.

The order of the perturbation equations is too large for a control implementation, and therefore some reduction is necessary. To this end, the elastic motion can be expanded into a series consisting of premaneuver vectors acting as admissible vectors. These premaneuver vectors correspond to a state of equilibrium prior to the maneuver, which may be characterized by either rest or steady rotation. Hence, a set of the Krylov vectors are generated in order to account for the

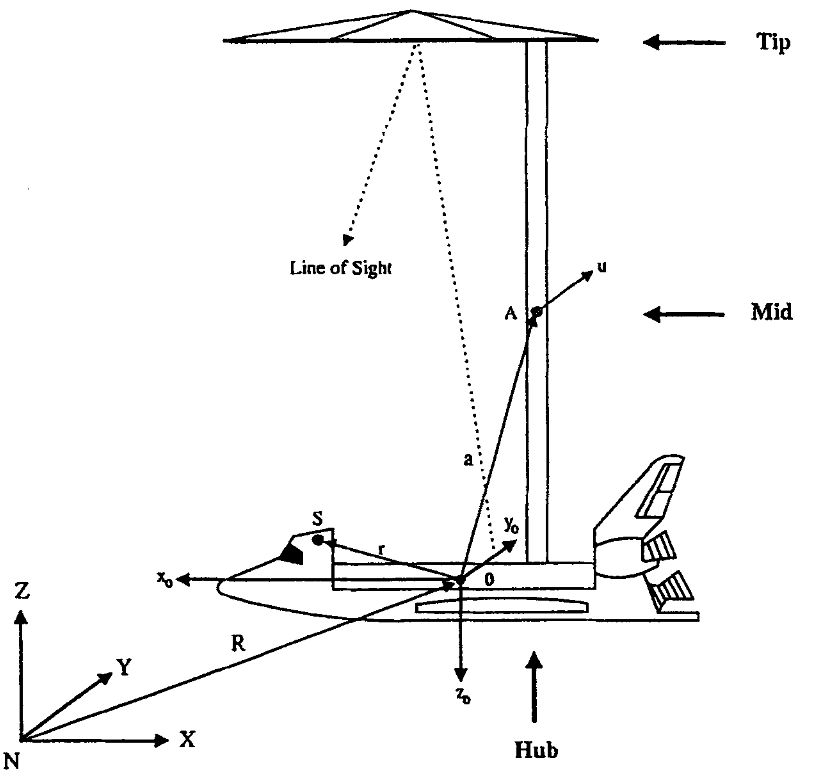

Fig. 1 Earth-orbiting spacecraft.

elastic modes which are excited by external inputs. By using these Krylov vectors which account for damping effects in the system, a more efficient model reduction is possible than by using eigenvectors. The task of numerical simulation can be carried out conveniently by means of the new model reduction algorithm.

The proposed FDKV algorithm is applied to the perturbated equations of motion of the SCOLE system in either rest or steady rotation. The premaneuver Krylov vectors correspond to a state of equilibrium prior to the maneuver, which may be characterized by either rest or steady rotation.

The perturbated equations of motion of the SCOLE system can be rewritten as

$$M\ddot{x}(t)+G(t)\dot{x}(t)+(K_0+K_t(t))x(t)=F^*u(t) \tag{13}$$

where K_t consists of the time-varying terms of the stiffness matrix and K_0 contains the constant stiffness terms. The proposed algorithm can be used for the perturbed equations of motion since the coefficients of the gyroscopic and stiffness matrices exist. The premaneuver equations of motion at rest can be expressed as

$$M\ddot{x}(t)+C\dot{x}(t)+K_0x(t)=F^*u(t) \tag{14}$$

where a Rayleigh damping term (Joo, et al., 1989) $C=\alpha M+\beta K$ is added in Eq. (14). Note that the elements of the damping term in rigid-body states are assigned zeros. The displacement vector of the premaneuver problem can be approximated by a linear combination of the iteratively generated Krylov vectors as follows:

$$x(t)=Qz(t) \tag{15}$$

where $z(t)\in\mathbf{R}^{p\times 1}$ is in Krylov space. Inserting Eq. (15) into Eq. (14) and premultiplying by Q^T, we obtain

$$\bar{M}\ddot{z}(t)+\bar{C}\dot{z}(t)+\bar{K}_0z(t)=Q^TF^*u(t) \tag{16}$$

where

$$\begin{aligned}\bar{M}&=Q^TMQ\\ \bar{C}&=Q^TCQ\\ \bar{K}_0&=Q^TK_0Q\end{aligned} \tag{17}$$

The premaneuver Krylov vectors have not decoupled the equations of motion. However, they take advantage of the efficient and accurate dynamic response of a small order system. We refer to Eq. (16) as being in *quasi-Krylov form*.

4. Numerical Evaluation

For the numerical simulation, the SCOLE model (Meirovitch and Quinn, 1987) is employed to illustrate the model reduction algorithm for semi-definite systems. A reduced-order model with the first 30 normal modes is used as a basis model due to the limitation of the time integrator for ordinary differential equation of MATLAB instead of using the full-order SCOLE model with 48 degrees of freedom. The damping matrix is assigned to the full-order model by using the Rayleigh damping for numerical simplicity and illustration. The damping formula is written as $C=\alpha M+\beta K$ where $\alpha=\beta=0.005$ were selected, but we know that the damping factor of space structures is very small. The portion of rigid-body coupling of the C matrix was nullified.

In Case 1, an actuator is located at the hub to the y-direction and a sensor is placed at the mast-tip to the y-direction. In the process of every FDKV iteration, two vectors will be generated due to the noncollocation of an actuator and a sensor. The model, denoted by FDKV (r2f1), is generated by consecutively using the spectrum shifting with $\sigma_r=5$ rad/sec for the first 4 elastic vectors and with $\sigma_f=10$ rad/sec for the last 2 elastic vectors. The model, denoted by (r1f2), is generated by consecutively using the same spectrum shifting parameters for the first 2 elastic vectors with σ_r and for the last 4 elastic vectors with σ_f. In Table 2, the damped eigenvalues are listed for comparison.

In the dynamic analysis for Case 1, the mast-hub is excited with non-zero initial velocity $\dot{y}_{tip}=1\ ft/\text{sec}\,(0.3048\text{ m/sec})$ for impulse response. In Fig. 2, the impulse responses of tip deflection of the four models are shown with respect to the undeformed structure. In the error plots in Figs. 3 and 4 with respect to the 30 basis model by taking the absolute values of impulse responses, it is clear that the impulse responses of the Krylov dependent vector sets, FDKV (r2f1) and FDKV

Table 2 Comparison of complex eigenvalues.

No.	12 Normal Vectors		12 FDKV Case 1 : r2f1		12 FDKV Case 1 : r1f2	
1	0		0		0	
2	0		0		0	
3	0		0		0	
4	0		0		0	
5	0		0		0	
6	0		0		0	
7	−9.3574e−2	6.0076e+0i	−9.3574e−2	6.0076e+0i	−9.3574e−2	6.0076e+0i
8	−1.0684e−1	6.4209e+0i	−1.0684e−1	6.4209e+0i	−1.0684e−1	6.4209e+0i
9	−8.0869e−1	1.7939e+0i	−8.0858e−1	1.7938e+1i	−8.0874e−1	1.7940e+1i
10	−1.6798e+0	2.5847e+1i	−1.6574e+0	2.5675e+1i	−1.2798e+0	2.2565e+1i
11	−5.0280e+0	4.4552e+1i	−5.3296e+0	4.5854e+1i	−6.1148e+0	4.0966e+1i
12	−1.3761e+1	7.2896e+1i	−1.7039e+1	8.0773e+1i	−1.7283e+1	8.1324e+1i

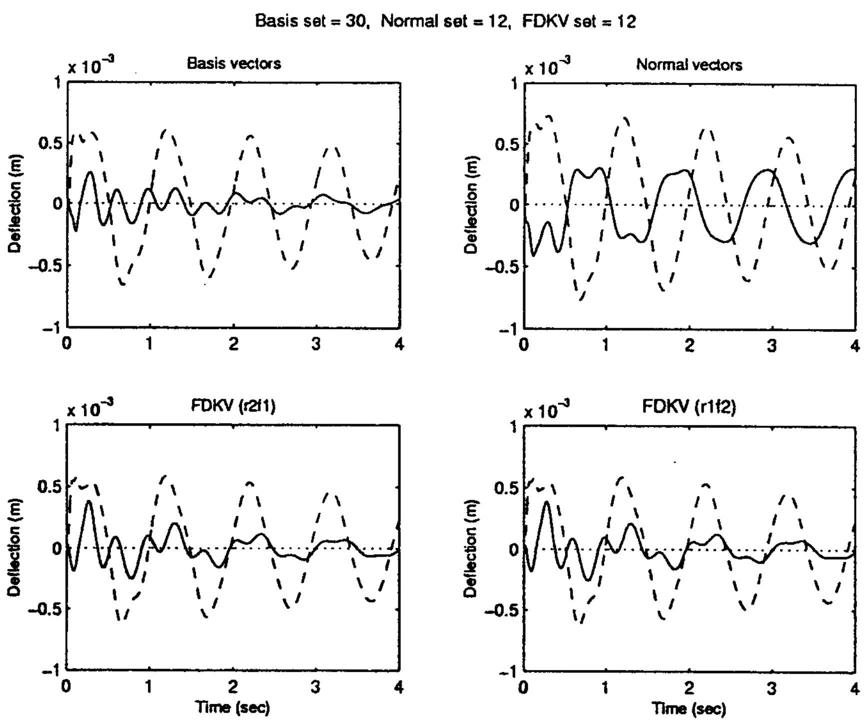

Fig. 2 Case 1: Impulse response of mast-tip of damped SCOLE model (solid: x-axis, dashed: y-axis, dot: z-axis).

(r1f2), are much more accurate than those of the 12 normal-mode set in both directions. It means that a small number of lower eigenmodes cannot represent the entire dynamic response, sufficiently. The impulse responses in axial direction, z axis, are quickly damped out for all the sets with very small differences. The impulse response of FDKV (r2f1) is more accurate than the one of FDKV (r1f2) and it implies that vectors related to lower frequencies are greatly participated in the dynamic response.

In Case 2, an actuator is located at the mid-

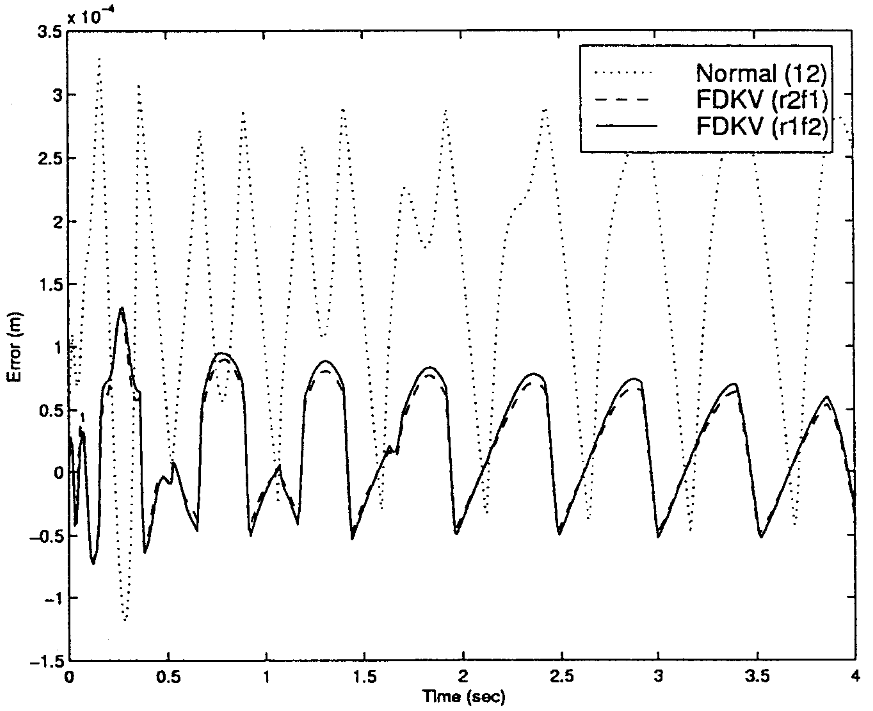

Fig. 3 Case 1: Deflection difference in x axis of mast-tip with respect to damped basis model.

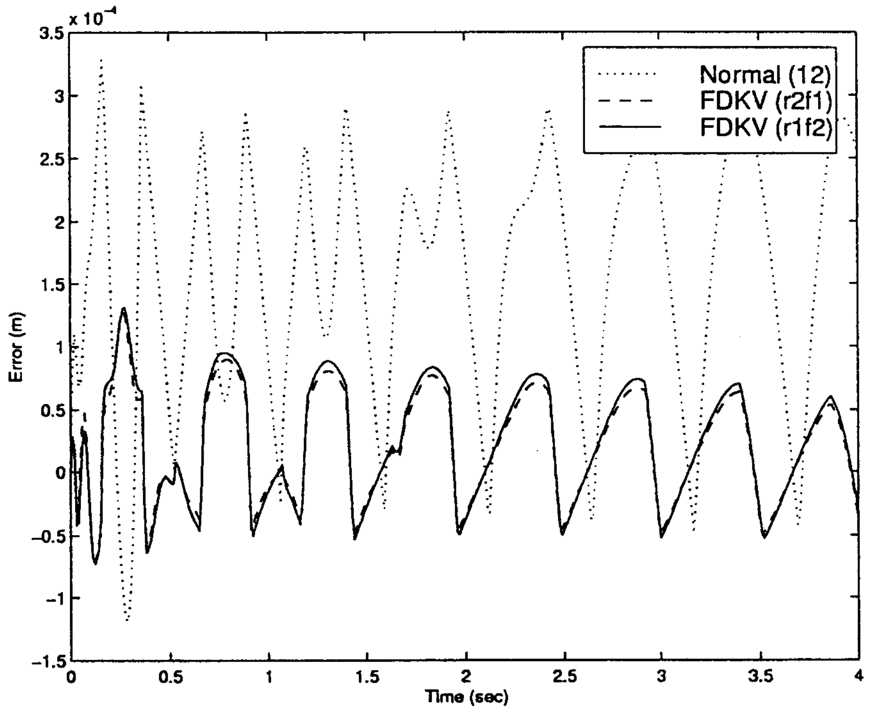

Fig. 4 Case 1: Deflection difference in y axis at mast-tip with respect to damped basis model.

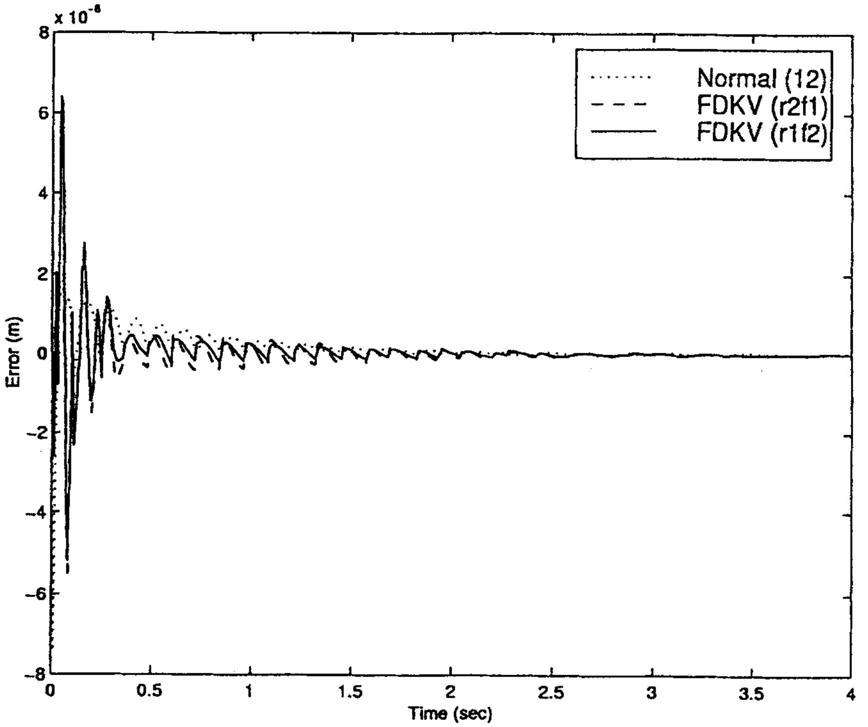

Fig. 5 Case 1: Deflection difference in z axis at mast-tip with respect to damped basis model.

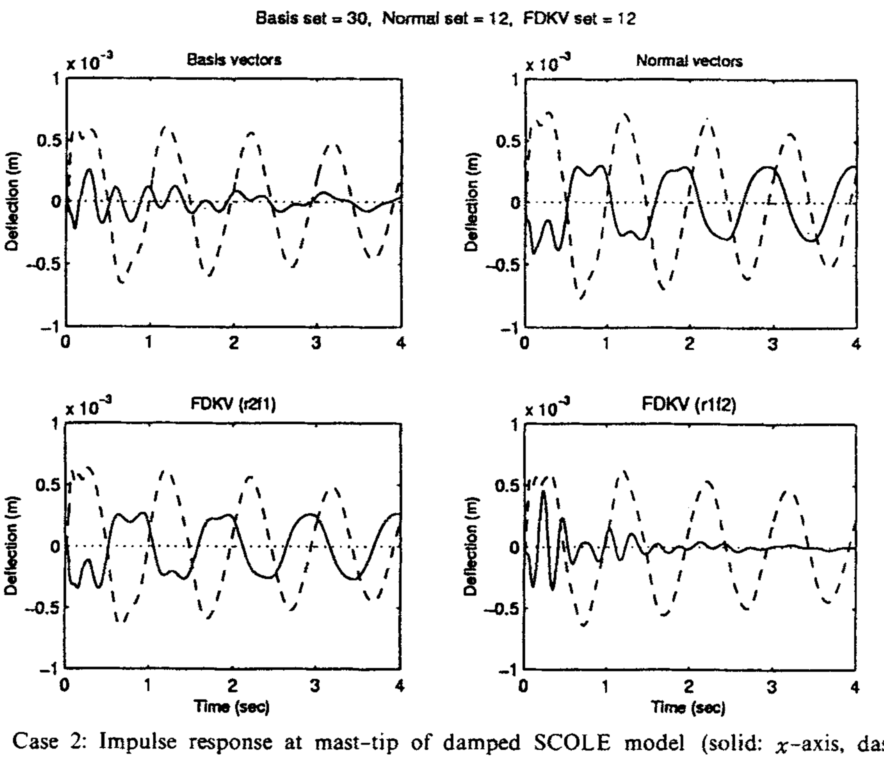

Fig. 6 Case 2: Impulse response at mast-tip of damped SCOLE model (solid: x-axis, dashed: y-axis, dot: z-axis).

Table 3 Comparison of complex eigenvalues.

No.	12 Normal Vectors		12 FDKV Case 1 : r2f1		12 FDKV Case 1 : r1f2	
1	0		0		0	
2	0		0		0	
3	0		0		0	
4	0		0		0	
5	0		0		0	
6	0		0		0	
7	−9.3574e−2	6.0076e+0i	−9.3574e−2	6.0133e+0i	−1.0672e−1	6.4170e+0i
8	−1.0684e−1	6.4209e+0i	−1.0684e−1	6.4209e+0i	−7.6933e−1	1.7496e+1i
9	−8.0869e−1	1.7939e+1i	−8.1057e−1	1.7960e+1i	−1.2798e+0	2.2565e+1i
10	−1.6798e+0	2.5847e+1i	−1.7221e+0	2.6169e+1i	−2.4221e+0	3.1014e+1i
11	−5.0280e+0	4.4552e+1i	−1.0791e+0	6.4798e+1i	−1.3660e+1	7.2638e+1i
12	−1.3761e+1	7.2896e+1i	−4.7995e+1	1.2997e+2i	−6.9972e+1	1.5196e+2i

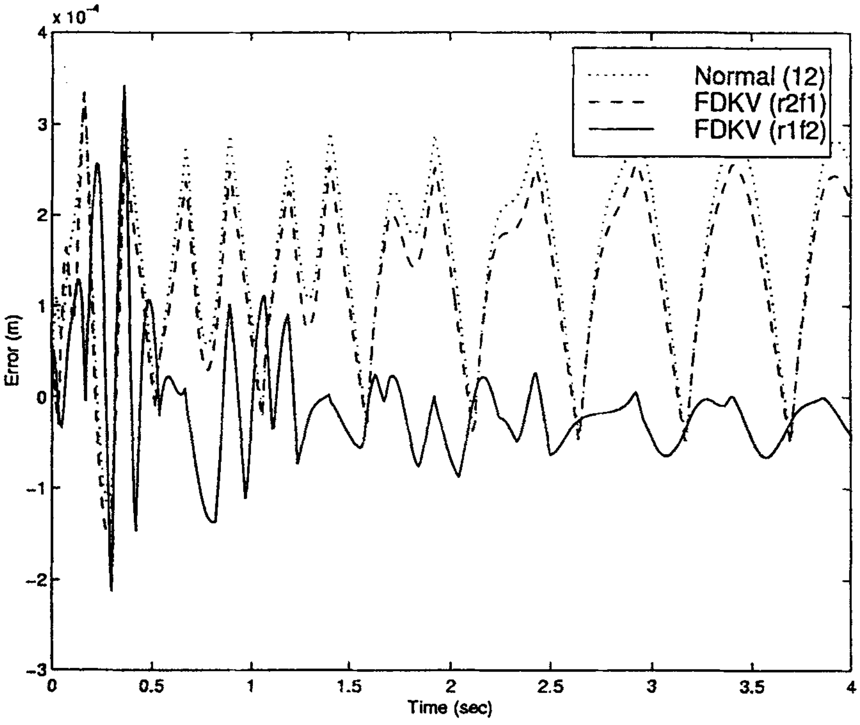

Fig. 7 Case 2: Deflection difference in x axis at mast-tip with respect to damped basis model.

point of the mast to the y-direction and a sensor is placed at the mast-tip to the y-direction. $\sigma_r = 5$ rad/sec and $\sigma_f = 30$ rad/sec are chosen for the accurate vector generation.

In the dynamic analysis for Case 2, the middle of the mast is excited with non-zero initial velocity $\dot{y}_{mid} = 1$ ft/sec (0.3048 m/sec) for impulse response. The impulse responses of the tip deflection of the four models are shown with respect to the undeformed structure in Fig. 6. In Figs. 7 and 8, the 2 FDKV sets show good agreement with the set of 30 normal-mode model. The FDKV (r2f1)

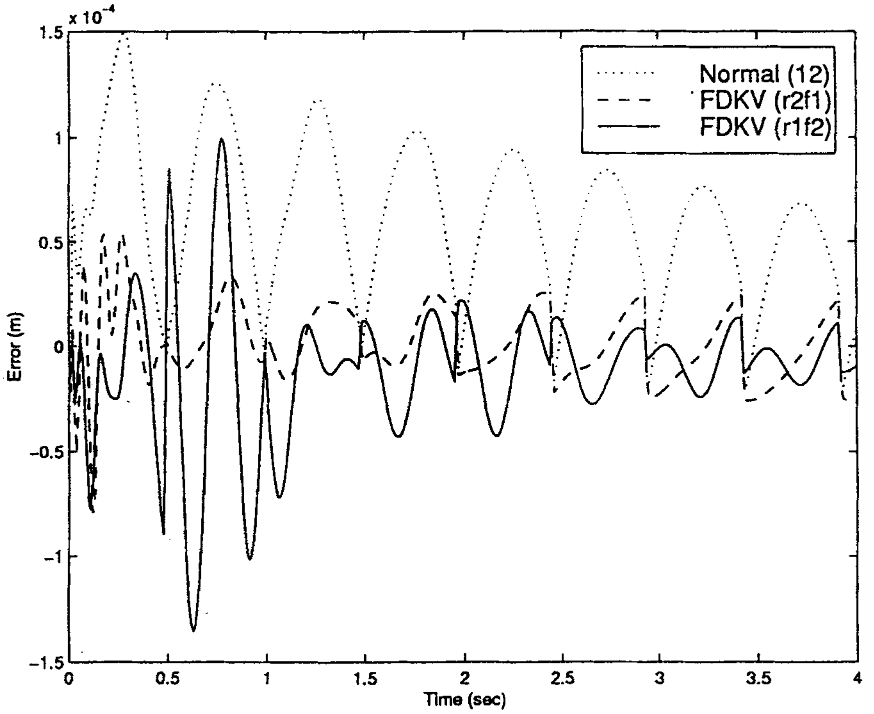

Fig. 8 Case 2: Deflection difference in y axis at mast-tip with respect to damped basis model.

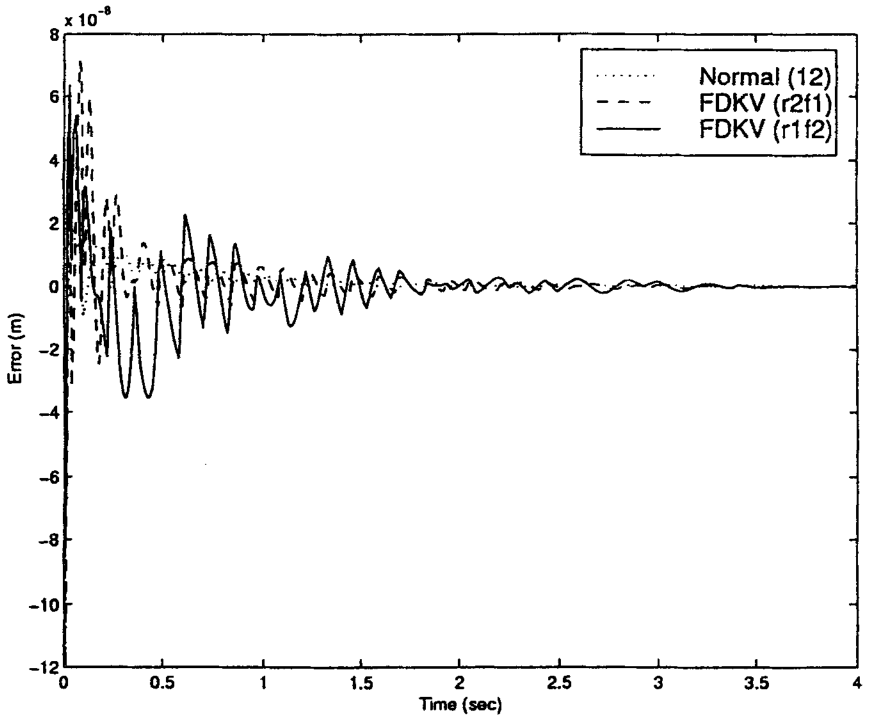

Fig. 9 Case 2: Deflection difference in z axis at mast-tip with respect to damped basis model.

shows better accuracy than the 12 normal-mode model in y-tip deflection. After 2 seconds, the FDKV (r1f2) shows better results than other models in both x and y directions. In Fig. 9, the deflection in z axis is almost negligible.

The FDKV algorithm for semi-positive definite systems is developed with the application to the SCOLE model. Based on the numerical analysis, the sets of FDKV vectors produced more accurate dynamic responses than the set of 12 normal modes did in two excitation cases with respect to the set of 30 normal modes. Therefore, we can say that low frequency modes are not adequate to represent the system response and therefore load dependent frequency modes must be strategically included to improve the dynamic response analysis. Note that eigenvectors are not always the best choice in dynamic analysis at least for the applications shown in the numerical analysis. By employing the FDKV algorithm, more accurate dynamic response can be obtained with a smaller number of vectors.

5. Conclusions

The algorithm of frequency dependent Krylov vectors (FDKV) was developed for semi-positive definite systems. In the nature of the FDKV procedure, an efficient vector set can be generated without increasing the system order and without using complex calculus, unlike the standard eigenproblem with damping effect. The superiority of systems transformed by the FDKV algorithm over those transformed by eigenvectors was demonstrated with the application to the quasi-Krylov equations of the SCOLE model. In the paper, pair of one-directional actuator and sensor was configured to generate the FDKV set for starting vector generation. For general cases, multi-directional locations of actuators and sensors could be chosen for realistic application of the FDKV algorithm. On the other hand, the FDKV produces further reduced-order model in the case of the collocation of each pair of actuator and sensor due to the dependency in starting vector selection.

As further researches, parametric studies are required to determine guidelines for selecting necessary number of vectors, optimum selection of σ_r and γ_f with regard to input loading and configuration of actuators and sensors. In addition, a warning mechanism is necessary because the shifted matrix will be nearly singular and numerical instability may arise.

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